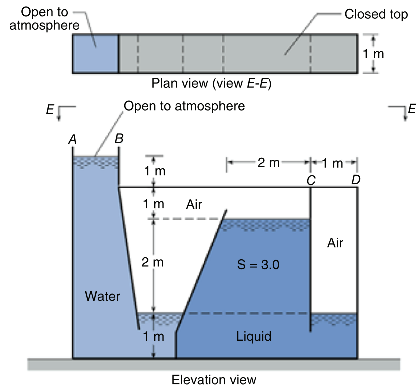
3.26: PROBLEM DEFINITION

Situation:

An odd tank contains water, air and a liquid.



Find:

Maximum gage pressure (kPa).

Where will maximum pressure occur.

Hydrostatic force (in kN) on top of the last chamber, surface CD.

Properties:

$\gamma_{\text{water}} = 9810\ \text{N/m}^3$.

PLAN

1. To find the maximum pressure, apply the manometer equation.
2. To find the hydrostatic force, multiply pressure times area.

SOLUTION

1. Manometer eqn. (start at surface AB; neglect pressure changes in the air; end at the bottom of the liquid reservoir)

$$\begin{aligned} 0 + 4 \times \gamma_{\text{H}_2\text{O}} + 3 \times 3\gamma_{\text{H}_2\text{O}} &= p_{\max} \\ p_{\max} &= 13\,\text{m} \times 9,810\,\text{N/m}^3 \\ &= 127,530\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p_{\max} = 127.53\ \text{kPa}}$$

Answer $\Rightarrow$ Maximum pressure will be at the bottom of the liquid that has a specific gravity of $S = 3$.

2. Hydrostatic force

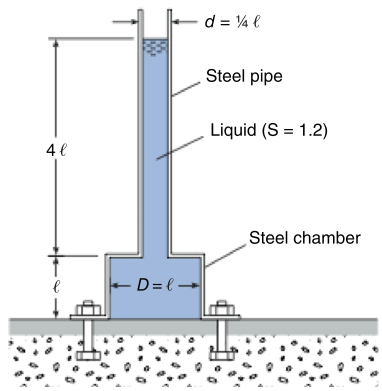
$$\begin{aligned} F_{CD} &= pA \\ &= (127,530\,\text{N/m}^2 - 1\,\text{m} \times 3 \times 9810\,\text{N/m}^3) \times 1\ \text{m}^2 \end{aligned}$$

$$\boxed{F_{CD} = 98.1\ \text{kN}}$$

3.27: PROBLEM DEFINITION

Situation:

A steel pipe is connected to a steel chamber.
$\ell = 0.75\,\text{m}$, $W = 2700\,\text{N}$.
$D_1 = 0.25\ell$, $z_1 = 5\ell$.
$D_2 = \ell$, $S = 1.2$.



Find:

Force exerted on chamber by bolts (N).

Properties:

$\gamma_{\text{water}} = 9810\,\text{N/m}^3$.

PLAN

Apply equilibrium and the hydrostatic equation.

SOLUTION

1. Equilibrium. (system is the steel structure plus the liquid within)

$$\text{(Weight of the steel)} = \begin{array}{l}\text{(Force exerted by bolts)} + \text{(Weight of the liquid)} + \\ \text{(Pressure force acting on the bottom of the free body)}\end{array}$$

$$F_B + W_{\text{liquid}} + W_s = p_2 A_2 \tag{1}$$

2. Hydrostatic equation (location 1 is on surface; location 2 at the bottom)

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma_{\text{liquid}}} + z_2 \\
0 + 5\ell &= \frac{p_2}{1.2\gamma_{\text{water}}} + 0 \\
p_2 &= 1.2\gamma_{\text{water}} 5\ell \\
&= 1.2 \times 9810 \times 5 \times 0.75 \\
&= 44,145 \text{ Pa}
\end{aligned}$$

3. Area

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi \ell^2}{4} = \frac{\pi \times (75\,\text{cm})^2}{4} = 0.442\,\text{m}^2$$

4. Weight of liquid

$$\begin{aligned} W_{\text{liquid}} &= \left(A_2\ell + \frac{\pi d^2}{4}4\ell\right)\gamma_{\text{liquid}} = \left(A_2\ell + \frac{\pi\ell^3}{16}\right)(1.2)\,\gamma_{\text{water}} \\ &= \left(\left(0.442\,\text{m}^2\right)(0.75\,\text{ft}) + \frac{\pi\,(0.75\,\text{m})^3}{16}\right)(1.2)\left(9810\,\text{N/m}^3\right) \\ &= 4877.1\,\text{N} \end{aligned}$$

5. Substitute numbers into Eq. (1)

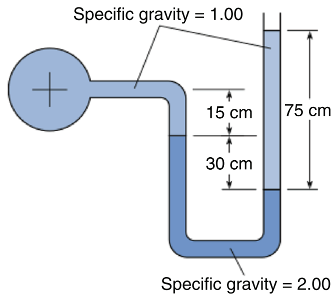
$$\begin{aligned} F_B + (4877.1\,\text{N}) + (2700\,\text{N}) &= \left(44145\,\text{N}/\text{m}^2\right)\left(0.442\,\text{m}^2\right) \\ F_B &= 11935\,\text{N} \end{aligned}$$

$$\boxed{F_B = 11935\ \text{N}}$$

3.45: PROBLEM DEFINITION

Situation:

A manometer is connected to a pipe which is going in and out of the page; pipe center is at the "+" symbol.



Find:

Determine if the gage pressure at the center of the pipe is:
(a) negative
(b) positive
(c) zero

PLAN

Apply the manometer equation and justify the solution using calculations.

SOLUTION

Manometer equation. (add up pressures from the pipe center to the open end of the manometer)

$$p_{\text{pipe}} + (0.15\text{ m})(9810\text{ N/m}^3) + (0.3\text{ m})(2 \times 9810\text{ N/m}^3) \quad (1)$$
$$-(0.75\text{ m})(9810\text{ N/m}^3) = 0$$

Solve Eq. (1) for the pressure in the pipe

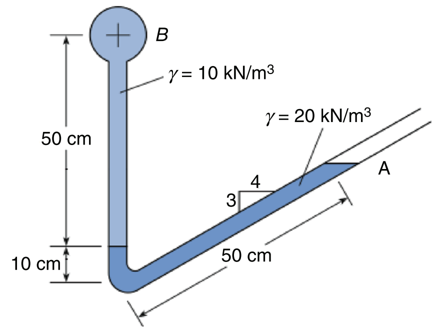
$$p_{\text{pipe}} = (-0.15 - 0.6 + 0.75)\text{ m }(9810\text{ N/m}^3) = 0$$

$$\boxed{p\,(\text{center of pipe}) = 0.0\,\text{N/ m}^2}$$

3.48: PROBLEM DEFINITION

Situation:

A tube (manometer) is connected to a pipe.



Find:

Pressure at the center of pipe B in units of kPa gage.

Properties:

$\gamma_1 = 10\,\mathrm{kN/m^3}$, $\gamma_2 = 20\,\mathrm{kN/m^3}$.

PLAN

Apply the manometer equation from point A (open leg of manometer, at the right side) to point B (center of pipe)

SOLUTION

Manometer equation

$$p_B = p_A + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i$$

were h_i denotes the vertical deflection in the i$^{\text{th}}$ section of the manometer

$$\begin{aligned} p_B &= (0\,\mathrm{Pa}) \\ &\quad + \left(0.30\,\mathrm{m} \times 20{,}000\,\mathrm{N/m^3}\right) \\ &\quad - \left(0.1\,\mathrm{m} \times 20{,}000\,\mathrm{N/m^3}\right) \\ &\quad - \left(0.5\,\mathrm{m} \times 10{,}000\,\mathrm{N/m^3}\right) \\ &= -1000\,\mathrm{Pa} \end{aligned}$$

$$\boxed{p_B = -1.00\ \text{kPa gage}}$$

REVIEW

Tip! Note that a manometer that is open to atmosphere will read gage pressure.

3.46: PROBLEM DEFINITION

Situation:

The boiling point of water decreases with elevation because p_{atm} decreases.
$z_1 = 2000\,\text{m}$, $z_2 = 4000\,\text{m}$.

Find:

Boiling point of water (°C) at z_1 and z_2.

Assumptions:

$T_{\text{sea level}} =$ 15ºC = 288K
Standard atmosphere.

Properties:

Table A.2: $R = 287\,\text{J/kg K}$.

PLAN

The pressure of boiling (p_{vapor}) corresponds to local atmospheric pressure.

1. Find the atmospheric pressure by calculating the pressure in the troposphere.
2. Find boiling temperature at 2000 m by interpolating in Table A.5.
3. Find boiling temperature at 4000 m by interpolating in Table A.5.

SOLUTION

1. Atmospheric pressure:

$$p = p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

$$= 101.3\,\text{kPa}\left[\frac{288\text{K} - 5.87\,\text{K/km}(z - z_0)}{288\text{K}}\right]^{g/\alpha R}$$

where

$$g/\alpha R = \frac{9.81\,\text{m/s}^2}{(5.87 \times 10^{-3})\,\text{K/m} \times 287\,\text{J/kg K}} = 5.823$$

So

$$p_{2000\,\text{m}} = 101.3\,\text{kPa}\left[\frac{288\text{K} - 5.87\,\text{K/km}(2.0\,\text{km})}{288\text{K}}\right]^{5.823} = 79.5\text{kPa}$$

$$p_{4000\,\text{m}} = 101.3\,\text{kPa}\left[\frac{288\text{K} - 5.87\,\text{K/km}(4.0\,\text{km})}{288\text{K}}\right]^{5.823} = 61.7\text{kPa}$$

2. Boiling temperature @ 2000 m.

$$T = 90\,^\circ\text{C} + \left(\frac{(79.5-70.1)\,\text{kPa}}{(101.3 - 70.1)\,\text{kPa}}\right)(10\,^\circ\text{C}) = 93.0ºC$$

$$\boxed{T_{\text{boiling, 2000 m}} \approx 93.0ºC}$$

3. Boiling temperature @ 4000 m.

$$T = 80\,^{\circ}\mathrm{C} + \left(\frac{(61.7 - 47.4)\,\mathrm{kPa}}{(70.1 - 47.4)\,\mathrm{kPa}}\right)(10\,^{\circ}\mathrm{C}) = 86.3\ ^{\circ}\mathrm{C}$$

$$\boxed{T_{\text{boiling, 4000 m}} \approx 86.3\ ^{\circ}\mathrm{C}}$$

3.51: PROBLEM DEFINITION

Situation:

A force due to pressure is acting on an airplane window.
Window is flat & elliptical. $a = 0.3\,\text{m}$, $b = 0.2\,\text{m}$.
$p_{\text{inside}} = 100\,\text{kPa}$, $z = 10\,\text{km}$.

Find:

Outward force on the window (in N).

PLAN

Find the force on the window by using $F = \Delta p A$. To find the pressure outside the window, the steps are

1. Calculate temperature at z = 10 km using the relevant equation for the troposphere.
2. Calculate pressure using the relevant equation for the troposphere.

SOLUTION

1. Temperature @ 10,000 m.

$$\begin{aligned} T &= T_0 - \alpha(z - z_0) \\ &= 288\text{K} - 5.87 \times 10^{-3}\,\text{K/m}(10000 - 0)\,\text{m} \\ &= 229.3\,\text{K} \quad -46.7^{\circ}\text{C} \end{aligned}$$

2. Atmospheric pressure @ 10,000 m.

$$\frac{g}{\alpha R} = \frac{(9.81\,\text{m/s}^2)}{(5.87 \times 10^{-3}\,\text{K/m})\,(287\,\text{J/kg}\cdot\text{K})} = 5.823$$

$$\begin{aligned} p &= p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R} \\ &= 101.3\,\text{kPa} \left[\frac{288\text{K} - (5.87 \times 10^{-3}\,\text{K/m})\,(10000 - 0)\,\text{m}}{288\text{K}}\right]^{5.823} \\ &= 26.86\,\text{kPa} \end{aligned}$$

3. Force on the window.

First, calculate the window area using the formula for an ellipse (from Figure A.1 in EFM9e)

$$A = \pi ab/4 = 3.1415 \times 0.3 \times 0.2 / 4 = 0.0471\,\text{m}^2$$

Now, calculate force

$$F = \Delta p A = (100 - 26.86)\frac{\text{kN}}{\text{m}^2}(0.0471\ \text{m}^2)$$

$$\boxed{F = 3.45\ \text{kN}}$$